PHYSICAL REVIEW E, VOLUME 64, 035301(R)

Intermittency of a passive tracer in the inverse energy cascade

Marie-Caroline Jullien,[1] Patrizia Castiglione,[1,2] and Patrick Tabeling[1]
[1]*Laboratoire de Physique Statistique, Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris, France*
[2]*INFM sezione Roma I, Piazzale Aldo Moro 2, 00185 Roma, Italy*

(Received 16 June 2000; published 14 August 2001)

We report an experimental study of the dispersion of a passive tracer in the two-dimensional inverse energy cascade, which shows that a nonintermittent velocity field can sustain a strongly intermittent concentration field. The experiment suggests the exponents of the intermittent concentration field saturate at large orders towards $\xi_\infty \sim 1.2$. These observations are in excellent agreement with a recent numerical work [A. Celani, A. Lanotte, A. Mazzino, and M. Vergassola, Phys. Rev. Lett. **84**, 2385 (2000)] and theoretical expectations [E. Balkovsky and V. Lebedev, Phys. Rev. E **58**, 5776 (1998); V. Yakhot, *ibid.* **55**, 329 (1997)].

DOI: 10.1103/PhysRevE.64.035301 PACS number(s): 47.27.Qb, 05.20.Jj

Hydrodynamic turbulence has resisted to theoretical assaults for decades. It seems the main difficulty for finding a decent solution to the problem is the presence of internal intermittency. Intermittency prevents a mean-field point of view from being adopted (at variance with many classical problems of physics) and jeopardizes the possibility of deploying standard perturbative methods. In an effort to handle simpler systems, Kraichnan considered the problem of the dispersion of a passive tracer by a Gaussian, self-similar, δ correlated velocity field, a situation further referred to as the Kraichnan model [1]. Solving this model has been a challenge for decades, and only recently two groups succeeded to work out analytical solutions [2,3]. We learn from their work that intermittent solutions naturally arise in Gaussian random velocity field. We now have a model where internal intermittency is fully understood and this piece of knowledge is currently stimulating new lines of thinking for the original turbulence problem. The fact that a Gaussian velocity field can sustain an intermittent concentration field has been shown theoretically in the context of a particular model [1], but so far we do not have any experimental indication this may happen in the real world. The dispersion studies performed until now have used non-Gaussian velocity fields and thus do not provide an answer. There is nonetheless an opportunity to tackle this question by studying the dispersion of passive tracers in the two-dimensional inverse cascade, since in this cascade, the velocity field is known to be (almost) Gaussian at all scales [4,5]. Such a study has been undertaken numerically [6,7] and intermittency of the concentration field has been obtained, confirming the (albeit loose) theoretical expectations. The objective of this letter is to convey experimental information on this remarkable physical situation.

The system we use has been described in [8] and in [4] for the preparation of the steady inverse cascade, and, for the sake of internal consistency, we recall here its main characteristics: the flow is generated in a square poly vinyl chloride (PVC) cell, 15 cm $\times$15 cm. The bottom of the cell is made of a thin (1 mm) glass plate, below which permanent magnets, $5\times8\times4$ mm in size are placed. In order to ensure two dimensionality, the cell is filled with two layers of NaCl solutions, 3-mm thick, with different densities $\rho_1 = 1030\ \mathrm{gl}^{-1}$ and $\rho_2 = 1060\ \mathrm{gl}^{-1}$, placed in a stable configuration, i.e., the heavier underlying the lighter. The interaction of an electrical current driven across the cell with the magnetic field produces local stirring forces. In the experiments we describe here, the experimental conditions of [4] have been exactly reproduced : the magnets are arranged so as the energy is injected, in the average, on a scale of 1.5 cm; the excitation is permanently maintained. In such conditions—as reported in [4]—the flow develops, after a short transient, an inverse energy cascade with Kolmogorov-Kraichnan scaling, $E(k) \sim k^{-5/3}$. In the stationary state, velocity correlation time measurements lead to $\tau_V \approx 4s$.

The passive scalar is a mixture of fluorescein and water [8], of density $\rho = 1030\ \mathrm{gl}^{-1}$, and diffusivity $\kappa = 10^{-6}\ \mathrm{cm}^2\,\mathrm{s}^{-1}$; in most cases, the colorant matches the upper layer density, and is vertically homogeneously spread across it, throughout the experiment. In several cases, we have used a lighter dye, of density $\rho = 1002\ \mathrm{gl}^{-1}$, i.e., 3% lighter than the upper layer, so as to operate with thin colorant sheets. The concentration field illuminated by ultraviolet light, is visualized using a 512×512 charge-coupled device (CCD) camera; we have checked the intensity be proportional to the scalar concentration. The images are stored and further processed. The overall spatial resolution for the concentration field is 0.2 mm. The ''effective'' Péclet number UL/κ_{eff} (where U and L are typical velocity and length scales) is typically on the order of 500. To inject the colorant, we first enclose a blob of dye within a cylinder, 5-cm diameter that is equal to the large scale of the inertial range, in the upper layer, then switch the electrical current on, wait for the transient state vanishes out, and eventually delicately remove the cylinder. In these experiments, the flow is statistically stationary, while the concentration field is in a freely decaying regime.

Additionally, in order to facilitate the analysis of the experimental observations, we have simulated the concentration field, using a method similar to [9,8]: the trajectories of 10^5 particles, located initially in a disk, 4-cm diameter, are calculated by integrating the experimentally measured velocity field. Coarsed graining is further applied to generate a concentration field. The simulation is thus designed to mimic the experiment. We will incorporate the results of this work in the discussion of the physical experimental results.

Figure 1(a) shows a typical evolution of a spot of fluorescein 5 cm in diameter, released in the center of the cell at time $t=0$. In the first few seconds, the blob boundaries develop small tendrils, while the overall shape is only slightly distorted [Fig. 1(a)-2]. In the range 5–15 s, the concentration

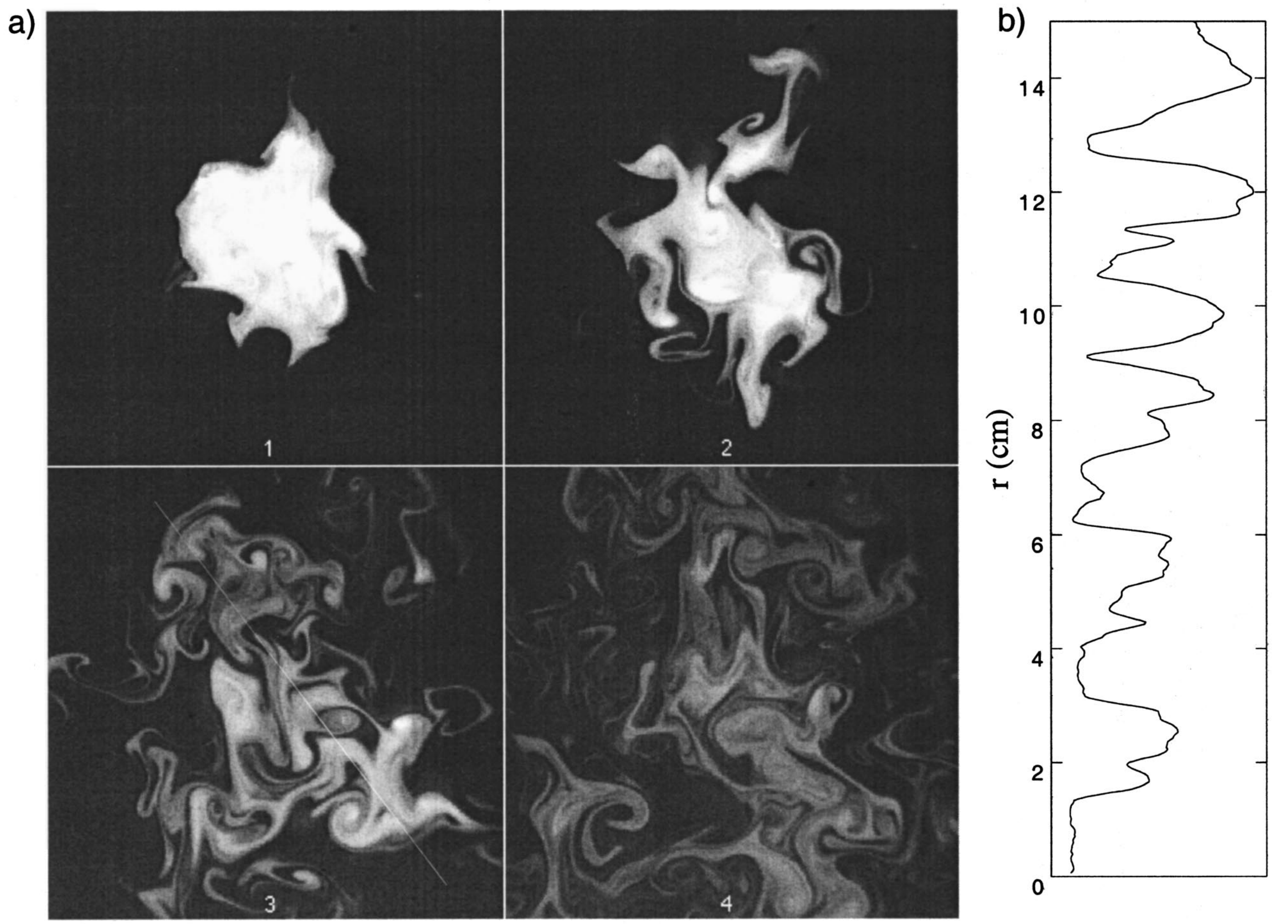

FIG. 1. (a) Time evolution of a blob of fluoresceine of density $\rho=1002\ \mathrm{gl}^{-1}$ in a 16 cm$\times$16 cm region, at times $t=0$, 4, 14, and 24 s. (b) Spatial profile of concentration along the straight line drawn at time $t=14$ s.

pattern seems to adopt a steady shape, well represented by Fig. 1(a)-3. It is formed by many tendrils and whorls, displaying various scales, comprised between a few millimeters (i.e., below the injection scale of energy) up to 1 or 2 cm. One may distinguish filaments, most of them highly curled, and of short extent. This situation contrasts with the dispersion of the same tracer by a smooth large scale velocity field, reported in [8], in which the typical filaments span a substantial fraction of the entire system. After 30 s [Fig. 1(a)-4], diffusion starts to come into play, and beyond 50 s mixing is achieved. A concentration profile along a line across the system is shown on Fig. 1(b), for $t=14$ s. The profile is formed of sharps cliffs and wide plateaus, indicating that high concentration increments are located on small scales.

We now discuss the inset of Fig. 2, which represents the evolution of the scalar dissipation $\chi=\langle(\nabla\theta)^2\rangle$ with time. At early times, χ increases, showing that gradients build up, as evidenced by Fig. 1(a)-2. At late times, diffusion becomes dominant and χ vanishes out. There is a maximum in between for $t=10$ s. Around the maximum, similarly as in stationary systems, a balance between the production of gradients by straining and their destruction by diffusion is approximately achieved. We may thus define a range of time (we consider here as lying between 5 and 15 s), within which the system can be treated as quasistationary.

Figure 2 displays the compensated scalar variance spectra $E_\theta(k)\times k^{5/3}$, now averaged over the entire quasistationary domain. The scalar variance spectra $E_\theta(k)$ is expected to evolve as $E_\theta(k)\approx\chi\epsilon k^{-5/3}$ [10], where ϵ is the energy transfer and χ is the mean scalar dissipation rate. The spectrum displays a range of wave number $k/2\pi$, comprised between 0.15 and 1 cm^{-1}, where a $k^{-5/3}$ law fairly holds. The range of scale in which the power law is observed is consistent with the scaling range developed by the energy cascade [4],

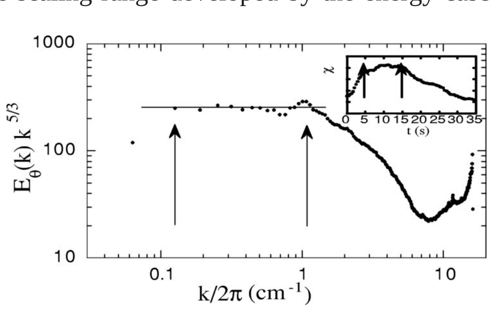

FIG. 2. Compensated variance scalar spectra $E_\theta(k)\times k^{5/3}$ of a pollutant of density $\rho=1002\ \mathrm{gl}^{-1}$. Straight line shows the inertial domain for which the scaling law is observed. An inset is shown of the time evolution of the scalar dissipation χ.

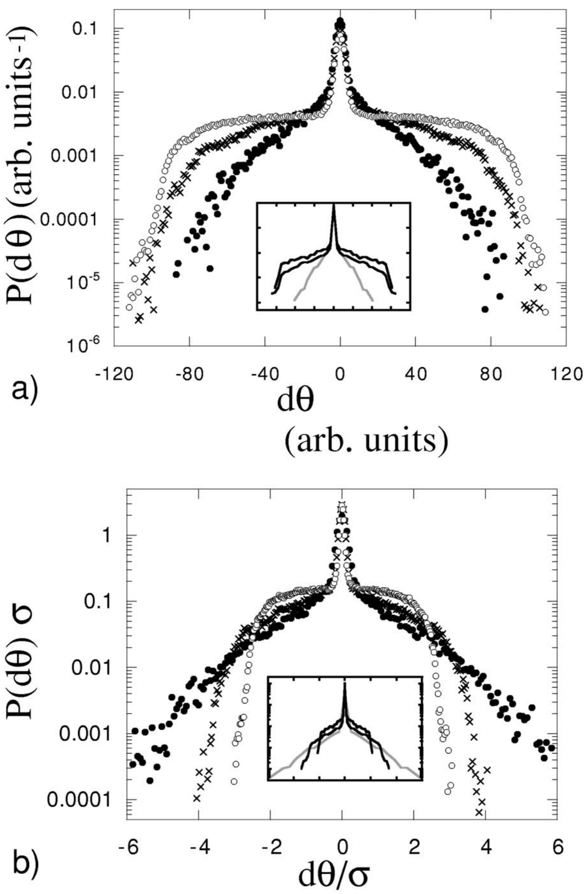

FIG. 3. $\rho = 1002 \ \text{gl}^{-1}$ (a) PDF of the concentration increments at time $t = 6$ s for three different scales. ●: $r = 0.3$ cm, ×: $r = 1.3$ cm, and ○: $r = 4$ cm. (b) Rescaled distributions of the concentration increments. Insets are the numerical results for the same scales.

but is slightly larger as in the experiments of [11]. At $k/2\pi = 1 \ \text{cm}^{-1}$, the spectrum shows a bump, and beyond this wave number, it drops. The bump on the concentration spectrum reveals the location of the injection scale of energy. Physically, the scalar are trapped into the numerous transient eddies that convey the energy into the system, and this generates a bump on the concentration spectrum. The drop at large wave numbers is indeed due to the action of diffusion, possibly enhanced by the shear across the layer, as previously noted for the Batchelor case.

We now turn to the measurement of the probability density functions (PDFs) of the concentration increments defined by

$$\Delta_r \theta(r) = (\theta(\mathbf{x} + \mathbf{r}) - \theta(\mathbf{x})),$$

where $\Delta_r \theta$ is the concentration increment measured on scale r. Figure 3(a) displays such PDFs, at $t = 6$ s in the quasistationary domain. These PDFs have the same behavior; a Gaussian hat at small scale, exponential tails, and then the PDFs drop , which signals the presence of a maximum increment in our system [see Fig. 1(b)]. Inset of Fig. 3(a) is the result for the numerical simulation for the same scales showing the same drop off effect below a maximum increment. Figure 3(b) displays the standardized (i.e., rescaled so as the

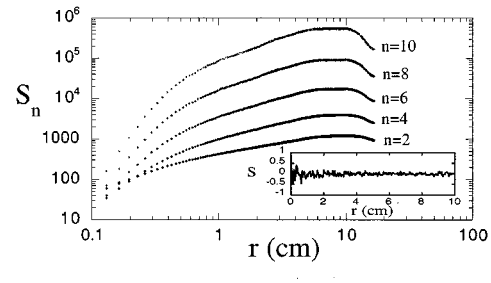

FIG. 4. Structure functions of orders up to ten, rescaled so as to compare in four decades. The passive scalar used has the density $\rho = 1002 \ \text{gl}^{-1}$. Inset is the skewness $S_3/S_2^{3/2}$ as a function of scale.

variance is unity) PDFs of the concentration increments, at $t = 6$ s. These PDFs appear nonself-similar and symmetrical. Our numerical study using simulated particles shows the same characteristics (see insets of Fig. 3). At this stage of the analysis, one obtains the concentration field is intermittent, since the rescaled PDF depend on the separation. We focus on this issue below.

As is traditionally done for turbulent signals, we measure the structure function of the concentration increments defined by

$$S_n(r) = \langle (\theta(\mathbf{x} + \mathbf{r}) - \theta(\mathbf{x}))^n \rangle.$$

Figure 4 displays such structure functions as a function of scale r, up to tenth order. The scaling range lies between 1 and 5 cm, and this may define the boundaries of the so-called "convective domain," i.e., a domain for that diffusion does not directly affect the characteristics of the concentration field. It is worthwhile to underline that power laws are well defined, a surprising fact for an experiment that hardly provides us with generously large ranges of scales. The odd structure functions are indistinguishable from zero at any ord der; to illustrate this point, the skewness $S_3(r)/S_2(r)^{3/2}$ is shown in the inset of Fig. 4; it fluctuates around zero, which is what we expect for isotropic systems.

The Kraichnan Obhukov Corrsin (KOC) theory yields the following scaling laws for all structure functions [10]:

$$S_{2n} \sim r^{\xi_{2n}} = r^{n\xi_2} = r^{2n/3}. \tag{1}$$

Our exponents, obtained by averaging the local derivatives of the curves of Fig. 4, within the convective domain, are

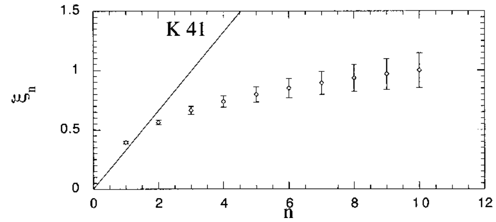

FIG. 5. Exponents of structure functions versus order. Straight line is the K41 scaling.

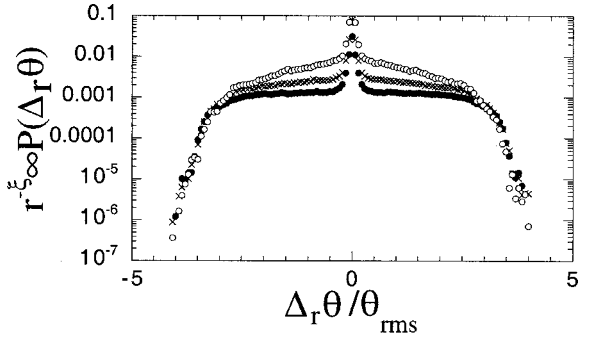

FIG. 6. Rescaled PDF $Q(\Delta_r\theta/\theta_{rms})=r^{-\xi_\infty}P(\Delta_r\theta)$, for three different scales in the convective domain. ●: $r=1.3$ cm, ×: $r=2.6$ cm, and o: $r=5$ cm.

shown on Fig. 5. The exponents fluctuate by 10% from an experiment to the other , so that it is difficult to propose accurate values characterizing the whole set of data we have. Nonetheless, the curve clearly deviates from KOC predictions. The anomaly, estimated to -0.38 ± 0.04 over the set of experiments we have, is in agreement with the numerical simulations [6], and turns out to be close to Kraichnan's model. At large orders, the rate of increase of the exponents seems to collapse and it is legitimate to ask whether the structure function exponents saturate at some value, say ξ_∞.

To address this issue, we use a method proposed in [6], and we investigate the rescaled PDF $Q(\Delta_r\theta/\theta_{rms})=r^{-\xi_\infty}P(\Delta_r\theta)$, where ξ_∞ is the exponent of saturation and $\theta_{rms}=\langle(\theta-\langle\theta\rangle)^2\rangle^{1/2}$. Because intermittency comes from strong events, only events $\Delta_r\theta$ much greater than θ_{rms} are taken into account, for these events, the tails of the PDFs $Q(\Delta_r\theta/\theta_{rms})$ are expected to be r independent in order to have saturation. Figure 6 shows that the tails of the PDFs $Q(\Delta_r\theta/\theta_{rms})$ collapse for an exponent of saturation of $\xi_\infty\sim1.2$, then the PDFs $Q(\Delta_r\theta/\theta_{rms})$ become r independent for strong events. This value for ξ_∞ is in good quantitative agreement with [6] and in good qualitative agreement with the theories [12,13], suggesting that ξ_n is n-independent for $n\rightarrow\infty$.

We have underlined that saturation comes from strong events $\Delta_r\theta\gg\theta_{rms}$. The probability $\mathcal{P}(\Delta_r\theta>\lambda\theta_{rms})$ to observe these strong events is given by

$$\mathcal{P}(\Delta_r\theta>\lambda\theta_{rms})=\int_{\lambda\theta_{rms}}^{\infty}P(\Delta_r\theta)d\Delta_r\theta\sim r^{\xi_\infty} \qquad (2)$$

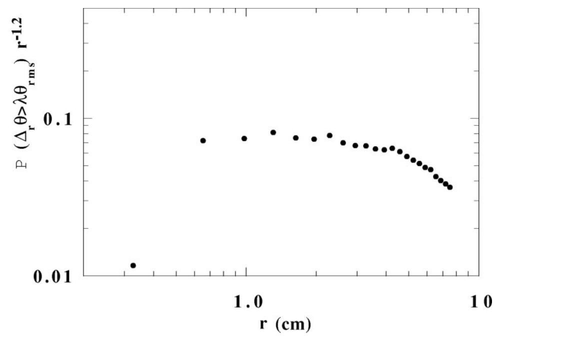

FIG. 7. Measure of $\mathcal{P}(\Delta_r\theta>\lambda\theta_{rms})$ for $\lambda=3$, compensated by $r^{-1.2}$.

On Fig. 7 the measure of $\mathcal{P}(\Delta_r\theta>\lambda\theta_{rms})$ for $\lambda=3$ is represented (the curves $Q(\Delta_r\theta/\theta_{rms})$ collapse for $3\theta_{rms}$) as a function of scale. The law (2) is observed for scales ranging from 0.7 cm to 4 cm, consistently with the power laws for the structure functions. This result provides an additional support to the saturation effect shown above.

To summarize, we have investigated the diffusion of a passive scalar in the inverse cascade of energy, for a free decaying blob, in a physical experiment. We have observed $k^{-5/3}$ spectrum, scale dependence for the distributions of concentration increments, power-law-like behavior for the structure functions, and agreement with saturation effects with a saturation exponent $\xi_\infty\sim1.2$. We thus bring here experimental evidence that a nonintermittent velocity field can sustain a strongly intermittent scalar field. The experiment suggests a saturation of the exponents at large order, a situation foreseen theoretically, but never observed. It is still hard to devise an experimental situation that exactly echoes the theoretical problem addressed by Kraichnan, since the model crucially assumes the velocity field be δ correlated in time, a condition unpleasant to substantiate experimentally. In our case, the correlation times of the concentration and the velocity fields are comparable; nonetheless, the trends we observe are similar in many respects.

This work was supported by the CNRS, the ENS, and the Universités Paris 6 and Paris 7. We acknowledge the support of the European Commission's TMR program, Contract No. ERBFMRXCT980175 "Intermittency in turbulent systems." We acknowledge valuable discussions with A. Mazzino and M. Vergassola.

[1] R. H. Kraichnan, Phys. Rev. Lett. **72**, 1016 (1994).
[2] K. Gawedzki *et al.*, Phys. Rev. Lett. **75**, 3834 (1995).
[3] M. Chertkov *et al.*, Phys. Rev. E **52**, 4924 (1995).
[4] J. Paret and P. Tabeling, Phys. Fluids **10**, 3126 (1998).
[5] L. M. Smith and V. Yakhot, Phys. Rev. Lett. **71**, 352 (1993).
[6] A. Celani, A. Lanotte, A. Mazzino, and M. Vergassola, Phys. Rev. Lett. **84**, 2385 (2000).
[7] M. Holzer and E. D. Siggia, Phys. Fluids **6**, 1820 (1994).
[8] M.-C. Jullien *et al.*, Phys. Rev. Lett. **85**, 3636 (2000).
[9] M.-C. Jullien *et al.*, Phys. Rev. Lett. **82**, 2872 (1999).
[10] U. Frisch, *Turbulence* (Cambridge University Press, Cambridge, England, 1995).
[11] Jayesh *et al.*, Phys. Fluids **6**, 306 (1994).
[12] E. Balkovsky and V. Lebedev, Phys. Rev. E **58**, 5776 (1998).
[13] V. Yakhot, Phys. Rev. E **55**, 329 (1997).